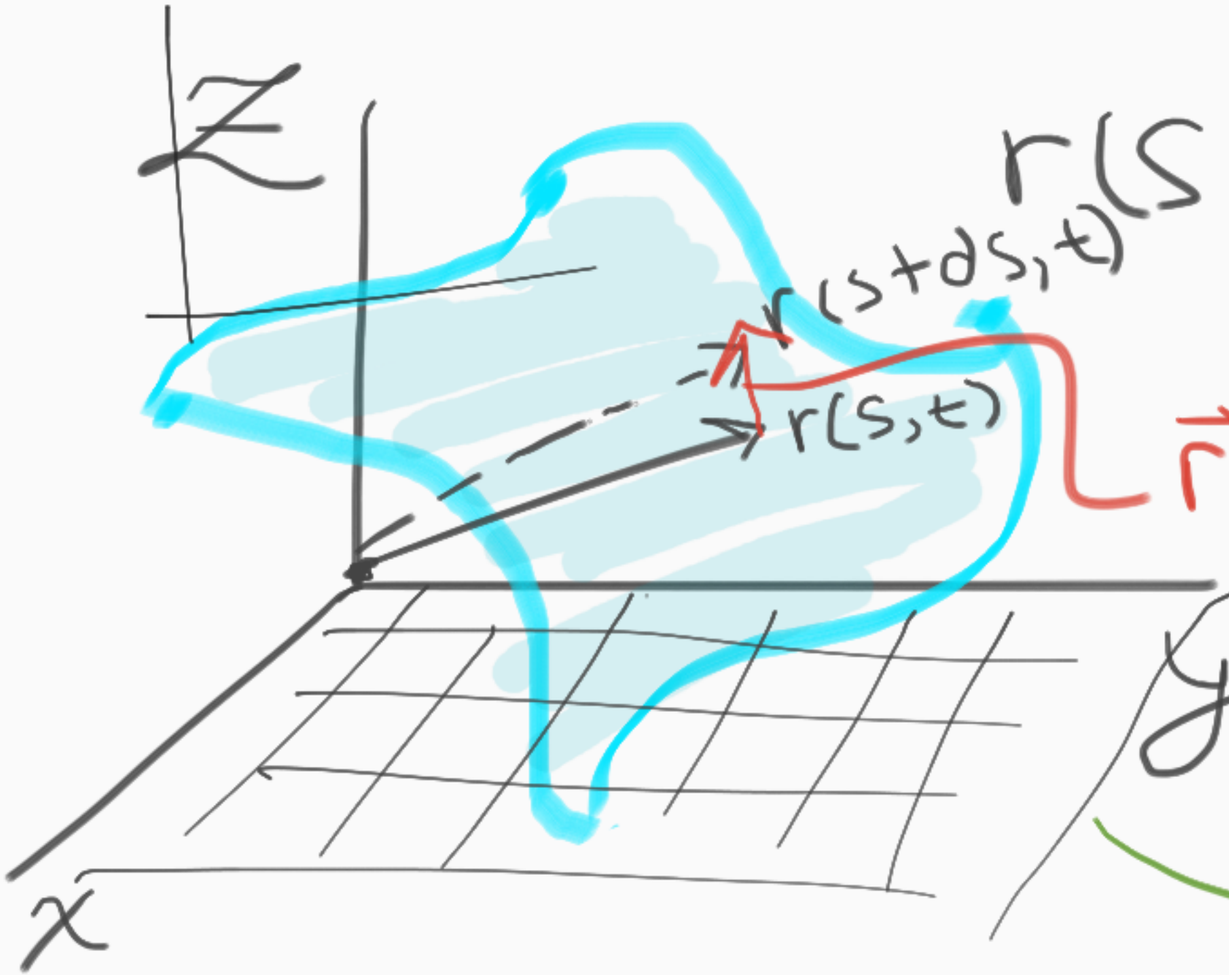
Intro to surfaceIntegrals



$$\vec{r}(s,t) = x(s,t)\hat{i} + y(s,t)\hat{j} + z(s,t)\hat{k}$$

$$\vec{r}(s+ds,t) - \vec{r}(s,t) = \frac{\partial \vec{r}}{\partial s}$$

$$\frac{\partial r}{\partial s} = \frac{\partial}{\partial s}x(s,t)\hat{i} + \frac{\partial}{\partial s}y(s,t)\hat{j} + \frac{\partial}{\partial s}z(s,t)\hat{k}$$

$$\frac{\partial r}{\partial t} = \frac{\partial}{\partial t}x\hat{i} + \frac{\partial}{\partial t}y\hat{j} + \frac{\partial}{\partial t}z\hat{k}$$

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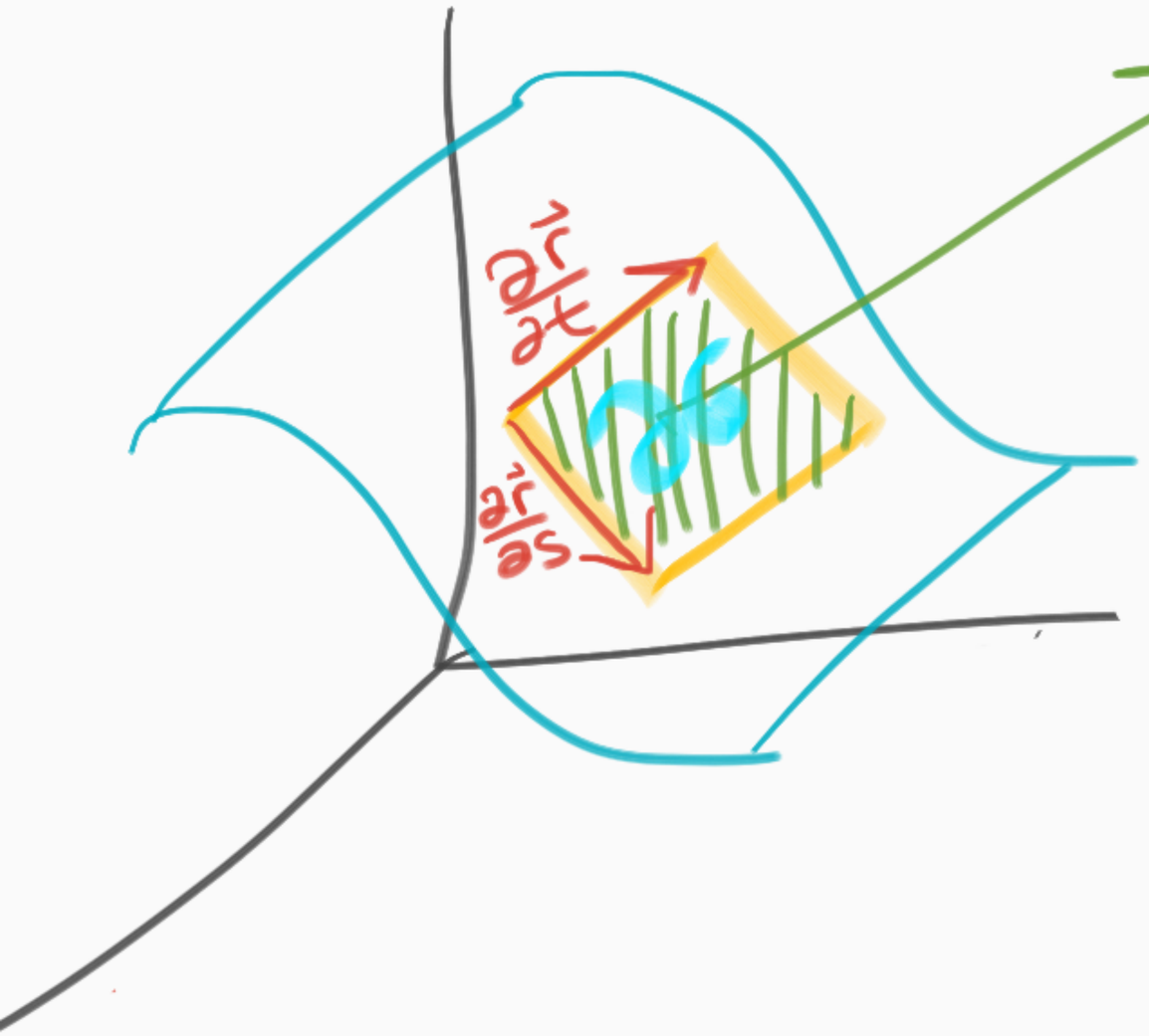
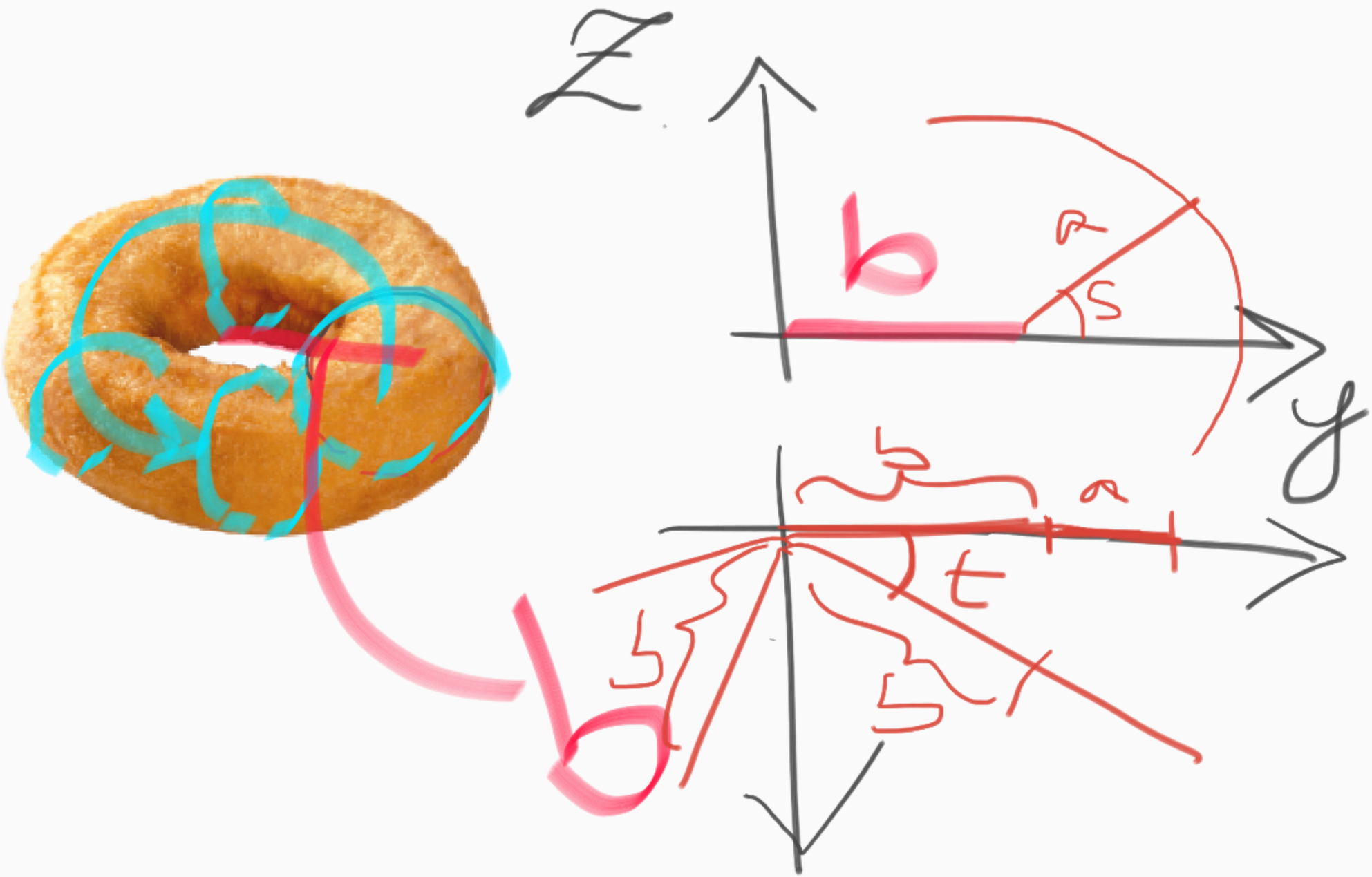


$$\left| \frac{\partial \vec{r}}{\partial s} \times \frac{\partial \vec{r}}{\partial t} \right|$$

$$\iint_{\Sigma} d\sigma = \iint_{A} \left| \frac{\partial \vec{r}}{\partial s} \times \frac{\partial \vec{r}}{\partial t} \right| ds\,dt$$

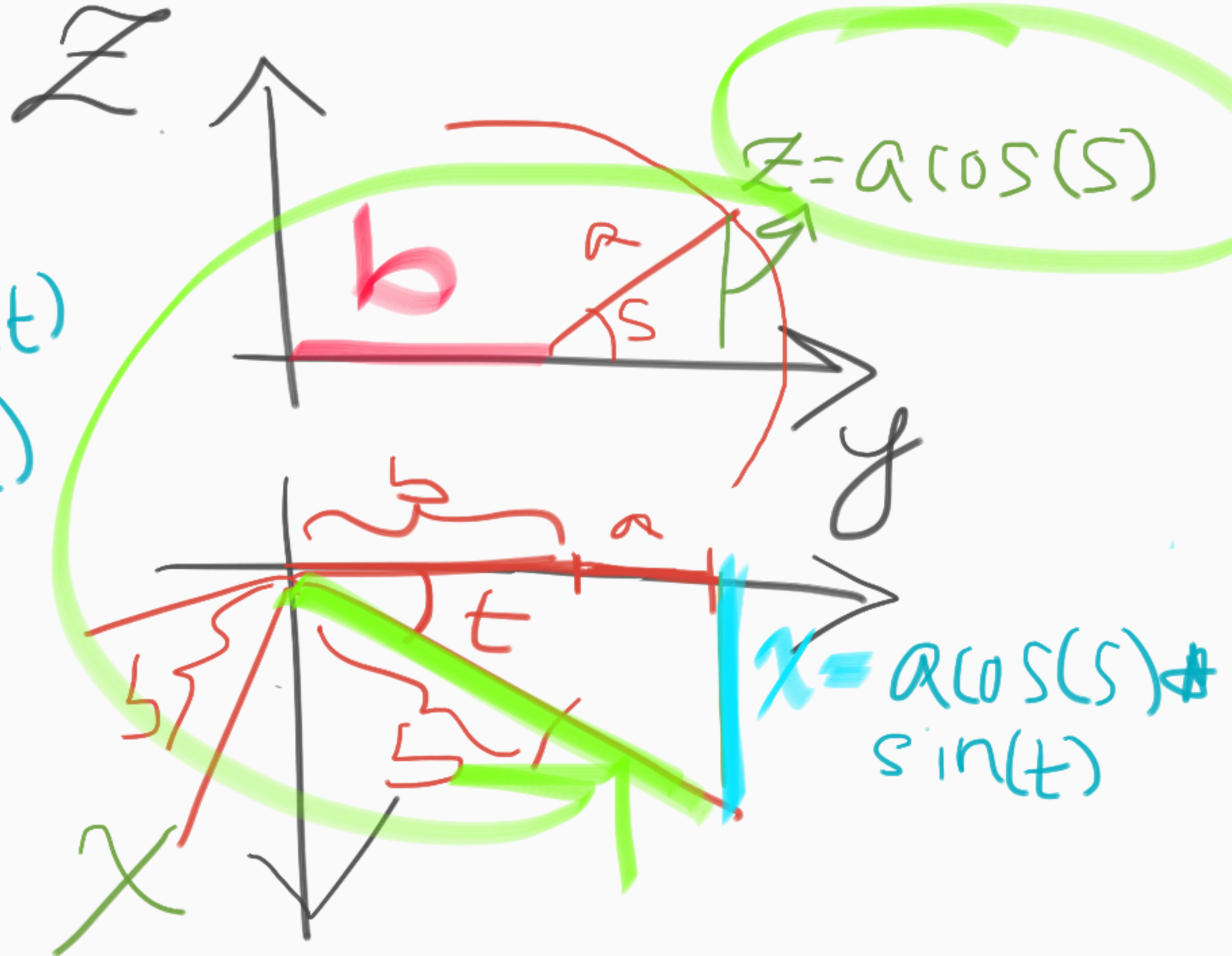
How can I parametrize a donut?


z
b
a
s
y
b
a
t
b
b

$$z = a\cos(s)$$

$$x = a\cos(s)\sin(t)$$

$$y = a\cos(s)\cos(t)$$



$$z = a\cos(s)$$

$$x = a\cos(s)\sin(t)$$

$$y = a\cos(s)\cos(t)$$

$$\vec{r}(s,t) = a\cos(s)\sin(t)\hat{i} + a\cos(s)\cos(t)\hat{j} + a\cos(s)\hat{k}$$

$$\frac{\partial \vec{r}}{\partial s} = a(-\sin(s)\sin(t))\hat{i} + a(-\sin(s)\cos(t))\hat{j} + (-a\sin(s))\hat{k}$$

$$\iint \left| \frac{\partial r}{\partial s} \times \frac{\partial r}{\partial t} \right| ds\,dt$$

$$\frac{\partial \vec{r}}{\partial t} = a\cos(s)\cos(t)\hat{i} + a\cos(s)(-\sin(t))\hat{j} + 0\hat{k}$$

$$\vec{r}(s,t) = a\cos(s)\sin(t)\hat{i} + a\cos(s)\cos(t)\hat{j} + a\cos(s)\hat{k}$$

$$\frac{\partial \vec{r}}{\partial s} = a(-\sin(s)\sin(t))\hat{i} + a(-\sin(s)\cos(t))\hat{j} + (-a\sin(s))\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a\cos(s)\cos(t) & -a\cos(s)\sin(t) & 0 \\ -a\sin(s)\sin(t) & -a\sin(s)\cos(t) & a\cos(s) \end{vmatrix}$$

$$-a^2\cos^2(s)\sin(t)\hat{i} - a^2\cos^2(s)\cos(t)\hat{j} + \left(-a^2\cos^2(t)\sin(s)\cos(s)\right)$$

$$+ \left(a^2\sin^2(t)\sin(s)\cos(s)\right)\hat{k}$$

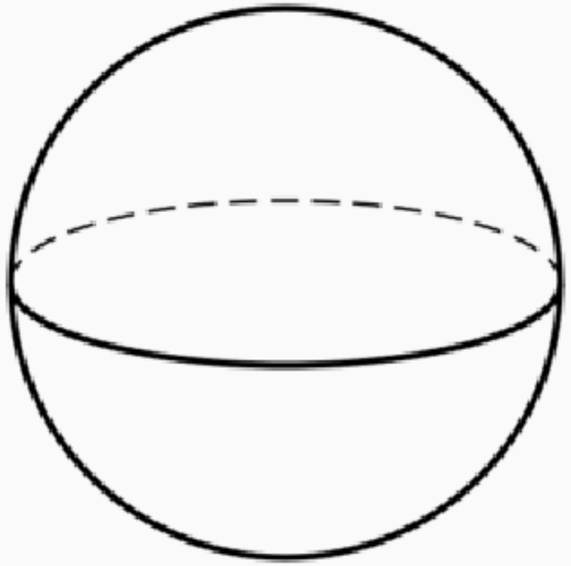
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a\cos(s)\cos(t) & -a\cos(s)\sin(t) & 0 \\ -a\sin(s)\sin(t) & -a\sin(s)\cos(t) & a\cos(s) \end{vmatrix}$$

$$-a^2\cos^2(s)\sin(t)\hat{i} - a^2\cos^2(s)\cos(t)\hat{j} + \left(-a^2\cos^2(t)\sin(s)\cos(s)\right)\hat{k}$$

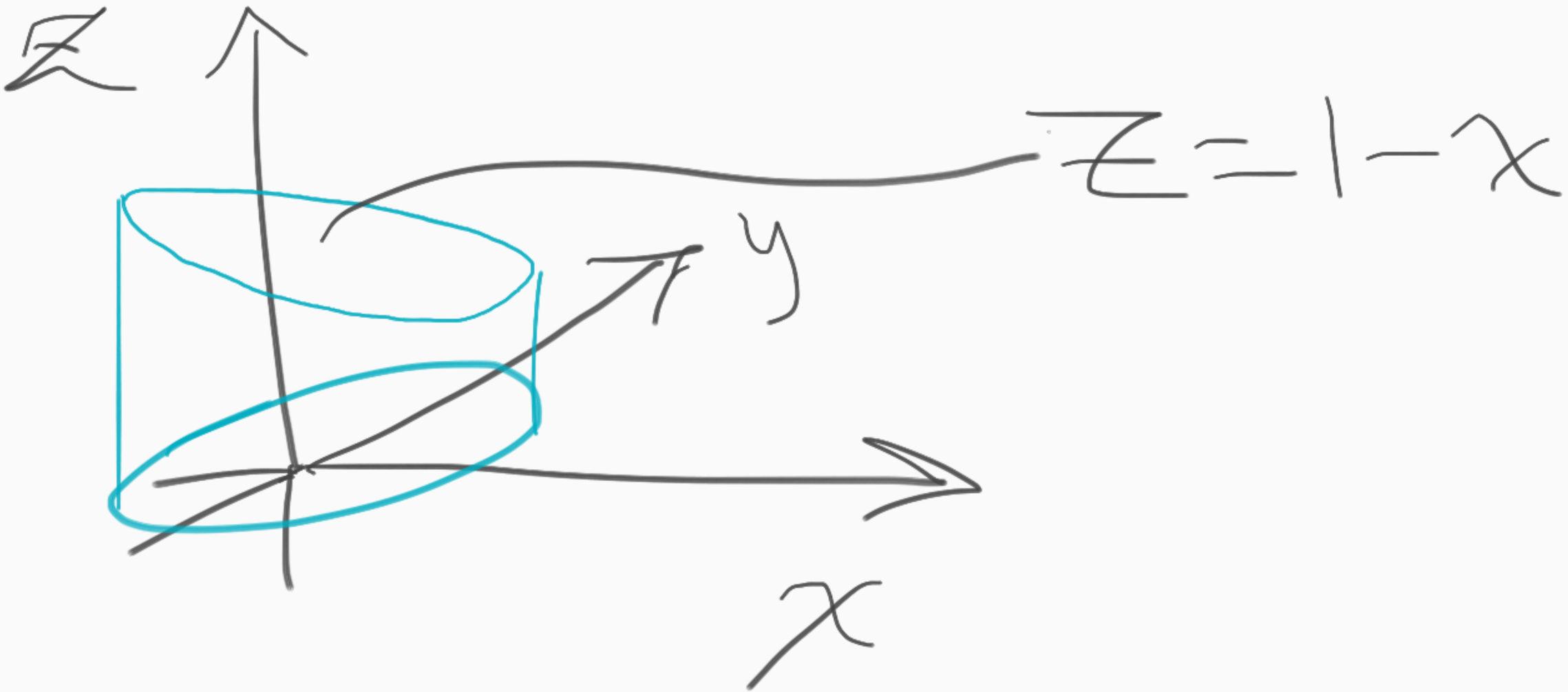
$$\sin(s)\cos(s)\left[-a^2\cos^2(t) + a^2\sin^2(t)\right]$$

$$\iint d\sigma$$

$$+ \left(a^2\sin^2(t)\sin(s)\cos(s)\right)\hat{k}$$



$$\iint_S y \, dS \quad S: x^2+y^2-z=0 \quad \Big\} 0 \le x \le 1, \; 0 \le y \le 2$$


z
z=1-x
y
x